Course Code:	24BSMA101
Course Name:	MATRICES AND CALCULUS

Module-I – MATRICES

Part-A

1. If $A^T = A$, then the matrix A is said to be _____

   a) Eigen values **b) Symmetric** c) Asymmetric d) Orthogonal

2. The characteristic equation of the matrix $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

   a) $\boldsymbol{\lambda}(\boldsymbol{\lambda}+3)(\boldsymbol{\lambda}+1) = 0$ b) $\boldsymbol{\lambda}(\boldsymbol{\lambda}-3)(\boldsymbol{\lambda}+1)=0$

   c)$(\boldsymbol{\lambda}+3)(\boldsymbol{\lambda}-1)= 0$ d)$\boldsymbol{\lambda}(\boldsymbol{\lambda}-3)(\boldsymbol{\lambda}-1)=0$

3. A square matrix A and its transpose A^T have --------eigen values

   a) complex b) interchange

   c) same d) different

4. If 1,2, and 3 are the eigen values A=$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ a & 0 & 2 \end{pmatrix}$ $find$ the value of 'a'?

   a) 3 b) 2 c) 0 d)1

5. Given A=$\begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$ Then eigen values of A^2 are

   a)1,9,9 b)1,1/9/1/4 c)-1, -3,2 d)1,9,4

6. The eigen values of a 4×4 matrix A are given as 2, -3,13 and 7. Then det(A) is

   a)-546 b)19 c)-25 d)0

7. If $\boldsymbol{\lambda}$ is a characteristic root of A, then $\boldsymbol{\lambda}$+k is a characteristic root of

   a) A+kI b) A-KI c) A d) I

8. The sum of the eigen values of the matrix $\begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{pmatrix}$ is

   a)0 b)2 c)-4 d)-2

9. If matrix A=$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$Then the eigen values of A^{-1} are

   a)1,2,2 b)1,1/2,1/2 c)3,2,0 d)1,2,0

10. Using Cayley Hamilton theorem for A=$\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}$ the value of $A^5-4A^4-7A^3+11A^2-A-10I$ is

a) A+3I b) A+5I c) A+2I d) A+I

11. If $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3$ are the eigen values of A^{-1} are

a) $\frac{1}{\lambda 1}, \frac{1}{\lambda 2}, \frac{1}{\lambda 3}$ b) none c) $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3$ d) $\frac{\lambda 1}{2}, \frac{\lambda 2}{2}, \frac{\lambda 3}{2}$

12. The sum of the eigen values of the matrix $\begin{bmatrix}1 & 2 & -2\\ 1 & 0 & 3\\ -2 & -1 & -3\end{bmatrix}$ is

a) 2 b) -2 c) 0 d) -3

13. The sum and product of the eigen values of a 3x3 matrix A whose characteristic equation is $\lambda^3 - 7\lambda^2 + 36 = 0$ is

a) 1,-7 b) 1,36 c) 7,36 d) 7, -36

14. Eigen values of the matrix A= $\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$ are 5 and 1. What are the eigen values of the matrix A^2.

a) 1 and 25 b) 4 and 9 c) 3 and 2 d) 6 and 4

15. A is a singular matrix of order 3 with eigen values 2 and 3. The third eigen value is

a) 2 b) 3 c) 0 d) 1

16. If the sum of the eigen values of the matrix $\begin{bmatrix}2 & 1 & 0\\ 0 & 4 & 3\\ 0 & 0 & k\end{bmatrix}$ is 5, then the value of k is

a) 1 b) -1 c) 2 d) 4

17. If 3 and 6 are two eigen values of $A = \begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$,then the eigen values of A^{-1} are

a) $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$ b) $\frac{1}{3}, \frac{1}{3}, \frac{1}{-2}$ c) $\frac{1}{3}, \frac{1}{6}, \frac{1}{-2}$ d) $\frac{1}{3}, \frac{2}{6}, \frac{1}{-2}$

18. If the eigen values of the matrix A of order 3x3 are 2,3 and 1, then the eigen values of adjoint of A are

a) 3,2,6 b) 3,2,1 c)1,2,2 d) 3,2,0

19. If the sum of two eigen values and trace of a 3x3 matrix A are equal, then $|A|$ is

a) 0 b)1 c)2 d)3

20. For a given matrix A of order 3, |A| = 32 and two of its eigen values are 8 and 2, then the sum of the eigen values is

a) 21 b)12 c)31 d)41

21. If a, b, c are the eigen values of the square matrix A, then the eigen values of A^T are

a) a, a, a b) a, b, $\frac{1}{c}$ c) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ d) a, b, c

22. If all the eigen values of $A \geq 0$ with at least one eigen values = 0, then Q.F is said to be_____

a) Indefinite b) Positive semi definite c) Definite d) Negative semi definite

23. If all the eigen values of $A \leq 0$ with at least one eigen values = 0, then Q.F is said to be_____

a) Indefinite b) Positive semi definite c) Definite d) Negative semi definite

24. If two Eigen values are positive and one is negative then nature of Q.F is _____

a) Indefinite b) Positive semi definite c) Definite d) Negative semi definite

25. Find the matrix of the Quadratic form $2x^2 + 2xy + 3y^2$

a) $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ *b)* $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ *c)* $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$

26. Determine the nature of the following quadratic form $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - 9x_3^2$ is

a) Indefinite b) Positive semi definite c) Definite d) Negative semi definite

27. Write the Quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{bmatrix}$

a) $2x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 4x_1x_3 - 4x_2x_3$

b) $2x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 4x_1x_3 + 4x_2x_3$

c) $2x_1^2 - 2x_2^2 - 3x_3^2 + 2x_1x_2 + 4x_1x_3 + 4x_2x_3$

d) $2x_1^2 + 2x_2^2 - 3x_3^2 + 2x_1x_2 - 4x_1x_3 - 4x_2x_3$

28. The index and signature of the quadratic form $x_1^2 + 2x_2^2 - 5x_3^2$ are _____

a) 2,2 b) 3, 3 c) 2,1 d)1,1

29. A homogeneous expression of second degree in several variable is called as _____

a) reduced form b) canonical form c) quadratic form d) All of the above

30. What is the nature of the quadratic form $x^2 + y^2 + z^2$ in four variables?

a) Indefinite b) Positive semi definite c) Definite d) Negative semi definite

Part-B

1. Define skew symmetric matrix

2. Define orthogonal matrix.

3. State Cayley-Hamilton Theorem.

4. If $\lambda_1, \lambda_2, \dots, \lambda_n$ are non-zero eigenvalues of square matrix of order n, then prove that $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_n}$ are eigenvalues of A^{-1}.

5. The product of two eigenvalues of the matrix A= $\begin{bmatrix} 6 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue of A.

6. One of the eigenvalues of $\begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$ is -9, find the other two eigenvalues.

7. If $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ is an eigenvector of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, find the corresponding eigenvalue.

8. If 3 and 15 are the two eigenvalues of A= $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, find | A | without expanding the determinant.

9. Check whether the matrix B = $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is orthogonal? Justify.

10. Write down the quadratic form corresponding to the matrix $\begin{bmatrix} 2 & 4 & 5 \\ 4 & 3 & 1 \\ 5 & 1 & 1 \end{bmatrix}$

11. Write down the matrix of the quadratic form

$2x_1^2 - 2x_2^2 + 4x_3^2 + 2x_1x_2 - 6x_1x_3 + 6x_2x_3$

12. Find the nature of the quadratic form

$x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$

Part-C

1. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{bmatrix}$.
2. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$
3. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

4. Verify Cayley -Hamilton theorem for A=$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}.
5. Verify Cayley -Hamilton theorem for the matrix A=$\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$ and hence find A^{-1} and A^4.
6. Use Cayley-Hamilton theorem to find the matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 - 5A^3 + 8A^2 - 2A + I$ if the matrix A=$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$
7. If A=$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, find A^n in terms of A and I.
8. Diagonalise the matrix A=$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ by an orthogonal transformation.
9. Diagonalize the matrix A=$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$by an orthogonal transformation.
10. The eigenvalues of a 3 x 3 real asymmetric matrix A corresponding to the eigenvalues 2,3,6 are $[1, 0, -1]^T$, $[1, 1, 1]^T$, $[-1, 2, -1]^T$respectively, find the matrix A.
11. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal transformation. Hence find rank, index, signature and nature of the quadratic form.
12. Reduce the quadratic form ${8x_1}^2 + 7{x_2}^2 + 3{x_3}^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ to the canonical form through an orthogonal transformation. Hence find rank, index, signature and nature of the quadratic form.
13. Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the canonical form through an orthogonal transformation. Also find a non- zero set of values (x_1, x_2, x_3) which makes this quadratic form is zero.